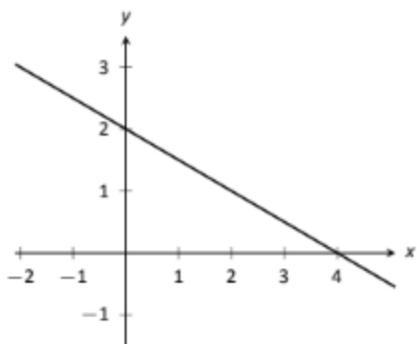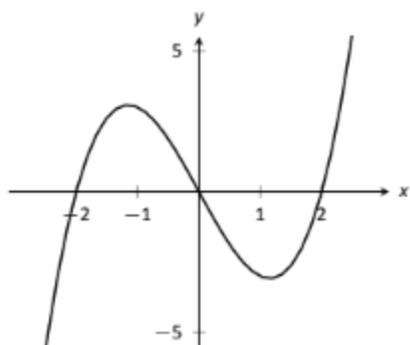# 6. Mixed Applications of Integrals and Derivatives

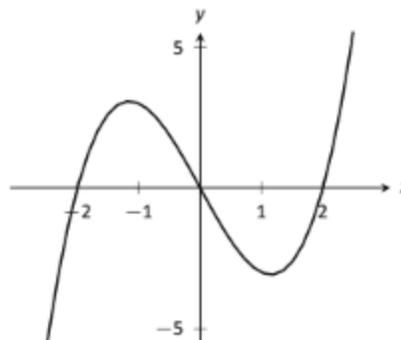## 6.1 Interpreting Graphs

In the following problems, a graph of a function $f(x)$ is given. Using the graph, sketch $f'(x)$
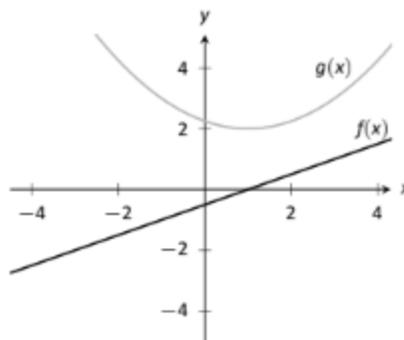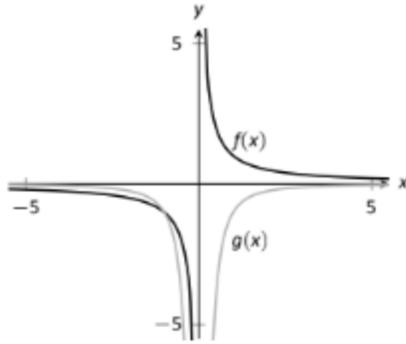

1.

2.

3.

In the following problems, identify which function is the derivative of the other?
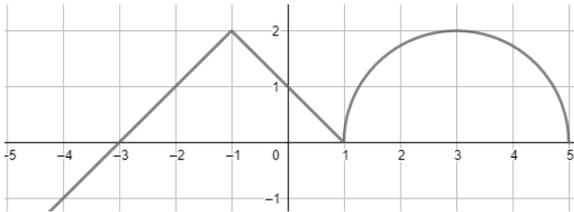

4.

In the following problem, a graph of a function $g(x)$ is given. Using the graph, answer the following questions:

(a) Where is $g(x) > 0$? $g(x) < 0$? $g(x) = 0$?

(b) Where is $g'(x) > 0$? $g'(x) < 0$? $g'(x) = 0$?

5.

6. Consider the following graph of $f'(x)$. It's constructed from two lines, and a semi-sphere with radius 2.

(a) If $f(-3) = 2$, compute the value of $f(1)$ and $f(5)$.
(b) What is the maximum and minimum value of $f(x)$ over the interval [-4,5]?
(c) Find the points of inflection for $f(x)$.
(d) Where is $f(x)$ increasing and decreasing?

7. Consider the following graph of $f'(x)$. It's constructed from a portion of the function $\sin(x)$ and two straight lines.
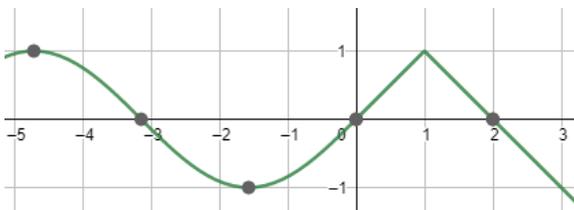
(a) If $f(0) = 4$, compute $f(-3)$ and $f(2)$.
(b) What is the maximum and minimum value of $f(x)$ over the interval $[-2\pi,3]$?
(c) Find the points of inflection for $f(x)$.
(d) Where is $f(x)$ increasing and decreasing?
(e) Where is $f'(x)$ not differentiable?
(f) If $g(x) = f(x)^2$, what is the value of $g'(0)$?

6.2 Motion Problems

Given the following velocity function $v(t)$ and initial value for an object moving in a straight line, find the displacement $x(t)$ and acceleration $a(t)$.

1. $v(t) = -32t + 20$, $x(0) = 0$

2. $v(t) = 10$, $x(0) = 10$

3. $v(t) = \cos t$, $x(0) = -5$

4. $v(t) = \sqrt{t} + t^2$, $x(0) = -2$

5. $v(t) = \dfrac{\ln t}{t}$, $x(1) = 10$

Given the following acceleration function $a(t)$ and initial values for an object moving in a straight line, find:
  (a) the displacement $x(t)$ and velocity $v(t)$.
  (b) When the object is speeding up and slowing down.
  (c) When the object changes directions.
  (d) The maximum position of the object.

6. $a(t) = -32$, $v(0) = 20$, $x(0) = 0$

7. $a(t) = 10t$, $v(0) = -50$, $x(0) = 100$

8. $a(t) = \sin t$ $0 \le t < 2\pi$, $v(0) = -1$, $x(0) = 5$

9. A particle moves along a straight line with velocity $v(t) = t^2 - 4$. It starts at time $t = 0$ at the origin, $x(0) = 0$. Find its position $x(4)$ at time $t = 4$, and the total distance traveled during the time interval [0,4]. Keep in mind that the distance traveled may or may not be the same as its final position.