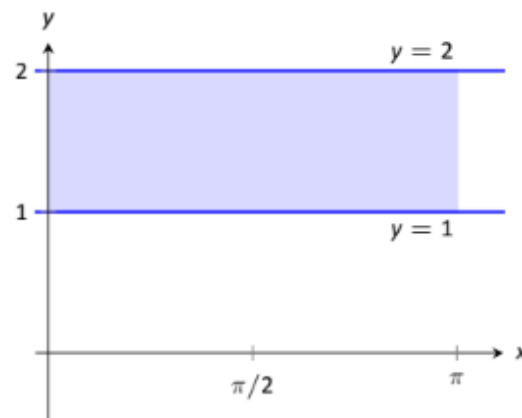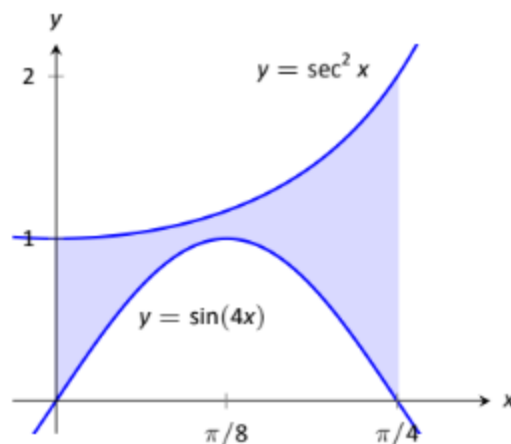# 5. Applications of Integration

5.1 Average Value

Find the average value of the function on the given interval.

1. $f(x) = \sin x$ on $[0, \pi/2]$

2. $y = x$ on $[0, 4]$

3. $y = x^3$ on $[0, 4]$

4. $g(t) = 1/t^2$ on $[1, 10]$

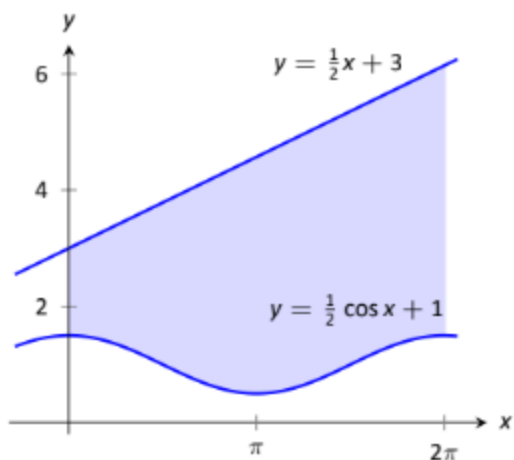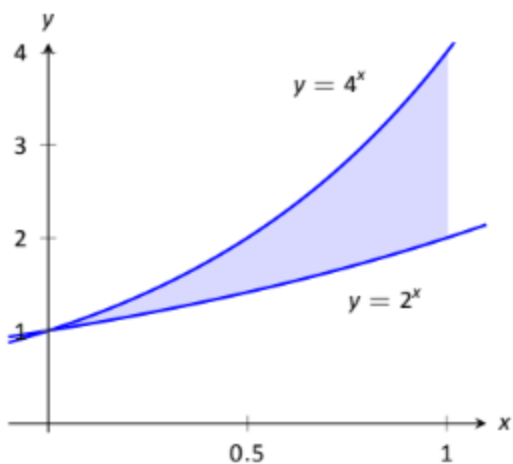5. Find the number k such that the average value of $f(x) = x^4$ on the interval $[-k,k]$ is equal to 1.
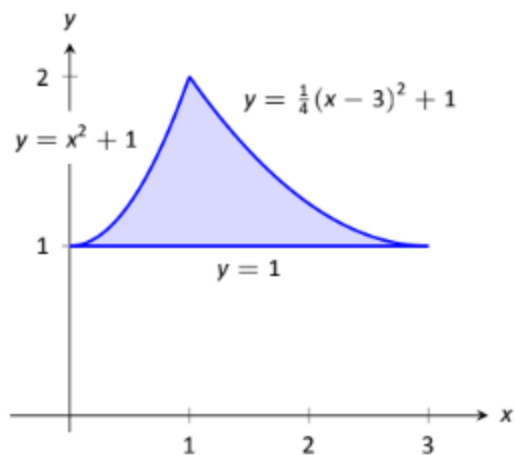
5.2 Area Between Curves

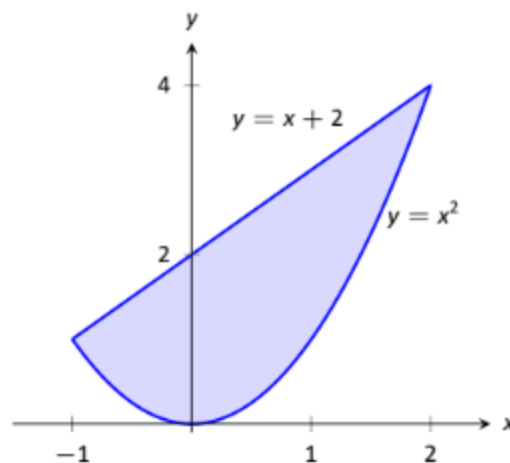Find the area between the shaded region:

2.

3.

1.

4.

9.

Find the total area enclosed by the functions f and g.

5. $f(x) = 2x^2 + 5x - 3, g(x) = x^2 + 4x - 1$

6. $f(x) = \sin x, g(x) = 2x/\pi$

7. $f(x) = x, g(x) = \sqrt{x}$

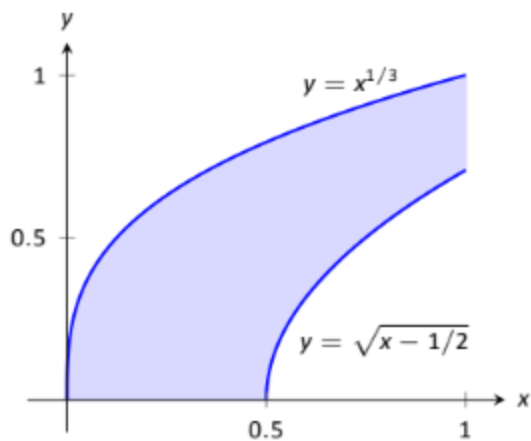8. $f(x) = \cos x, g(x) = \sin x$ (only include the area over one period)

10.

Compute the following areas by using:
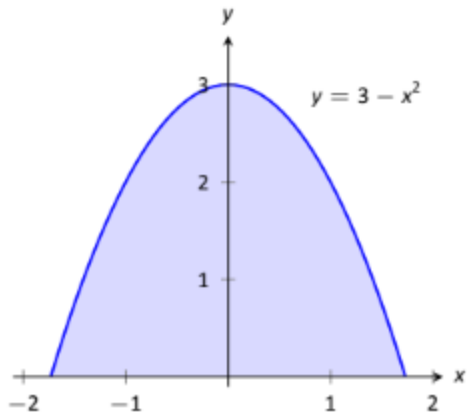   (a) An integral over dx
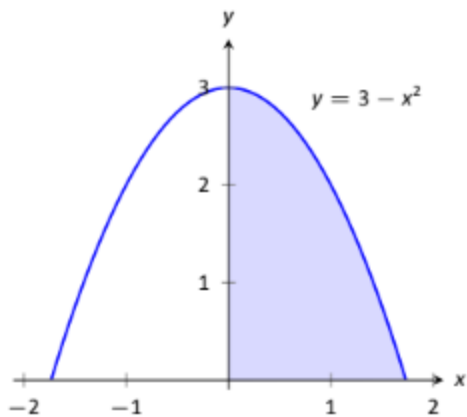   (b) An integral over dy
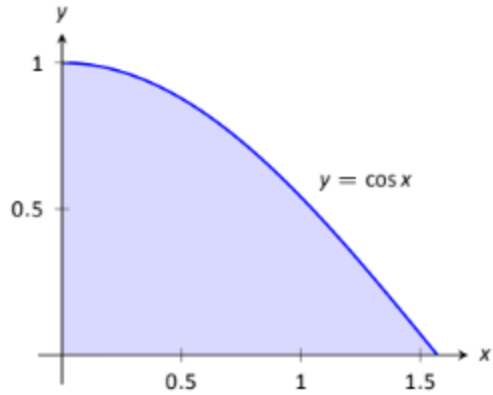

11.

5.3 Volume (Washer and Disk Method)

Use the washer or disk method to find the volume of the solid created when the shaded region is revolved around the x-axis.
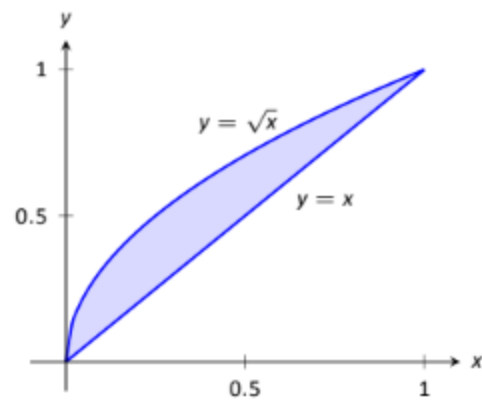

3.

1.

4.

2.

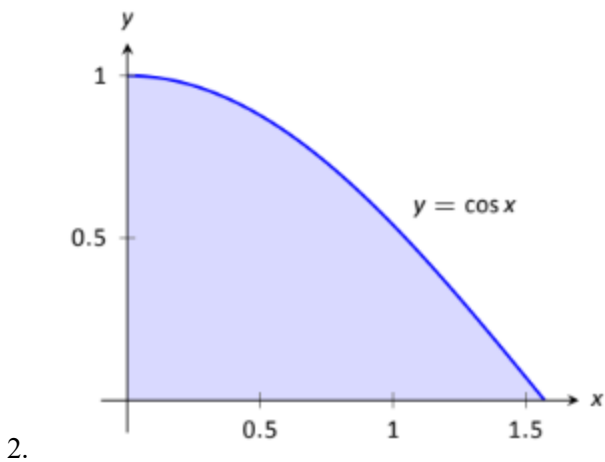Use the washer or disk method to find the volume of the solid generated by rotating the region about each of the given axes.

5. Region bounded by: $y = \sqrt{x}, y = 0$ and $x = 1$
   Rotate about:
   (a) The x-axis
   (b) $y = 1$
   (c) The y-axis
   (d) $x = 1$

6. The triangle with vertices $(1, 1), (1, 2)$ and $(2, 1)$
   Rotate about:
   (a) The x-axis
   (b) $y = 2$
   (c) The y-axis
   (d) $x = 1$

7. Region bounded by
   $y = 1/\sqrt{x^2 + 1}, x = -1, x = 1$ and the x-axis.

   Rotate about:
   (a) The x-axis
   (b) $y = 1$
   (c) $y = -1$

5.4 Volume (Shell Method)

Use the shell method to find the volume of the solid created when the shaded region is revolved around the y-axis.

1.

2.

(Note: Only setup the integral for this one)

Use the shell method to find the volume of the solid created when the shaded region is revolved around the x-axis.
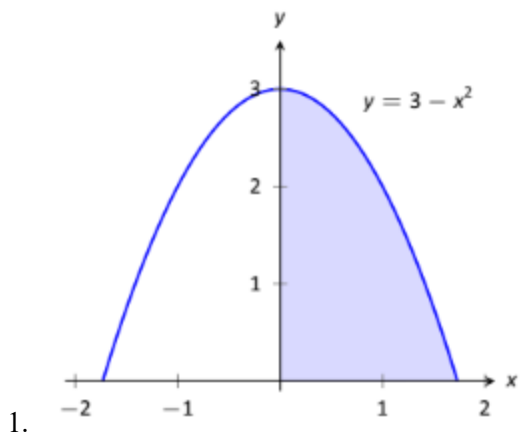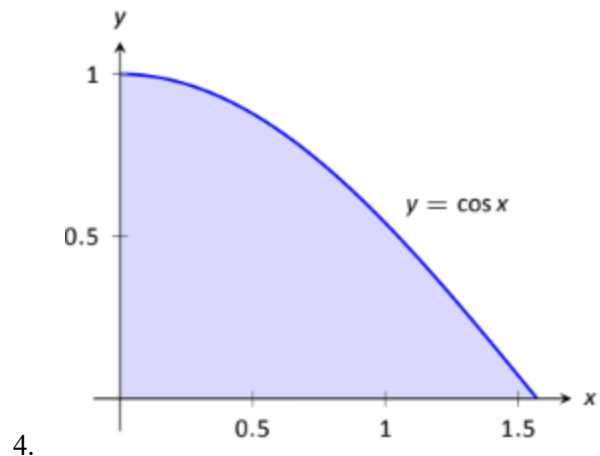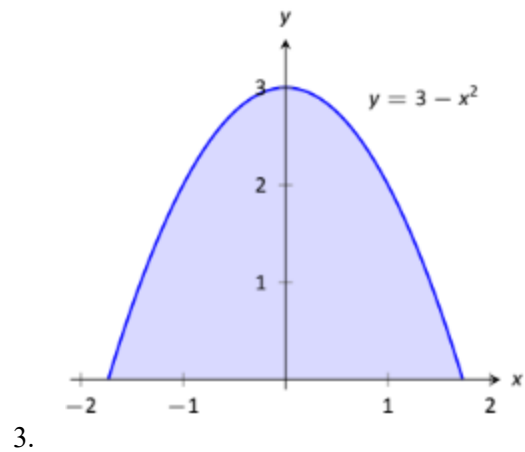

3.

4.

Note: Only setup the integral for this one.

Use the shell method to find the volume of the solid generated by rotating the region about each of the given axes.

5. Region bounded by: $y = \sqrt{x}, y = 0$ and $x = 1$
   Rotate about:
   (a) The x-axis
   (b) $y = 1$
   (c) The y-axis
   (d) $x = 1$

3. Consider the following region on the interval [0,1].

6. The triangle with vertices $(1,1), (1,2)$ and $(2,1)$
   Rotate about:
   (a) The x-axis
   (b) $y = 2$
   (c) The y-axis
   (d) $x = 1$

7. Region bounded by
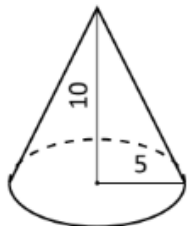 $y = 1/\sqrt{x^2 + 1}, x = -1, x = 1$ and the x-axis.
   Rotate about:
   (a) The x-axis
   (b) $y = 1$
   (c) $y = -1$

5.5 Volumes using Cross-Sectional Areas

Find the following volumes by finding the cross-sectional areas and integrating.

Find the volume if the cross-sectional areas perpendicular to the x-axis are:
a) Squares
b) Isosceles Right Triangles
c) Semispheres
d) Equilateral Triangles
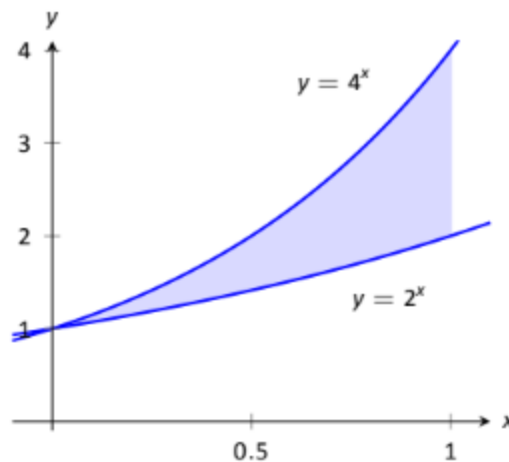
A right circular cone with height of 10 and base radius of 5.

1.

A right triangular cone with height of 10 and whose base is a right, isosceles triangle with side length 4.

2.